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MOTION OF AN ALMOST IDEAL GAS IN THE PRESENCE OF A STRONG POINT EXPLOSION

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S. I. ANISIMOV and O. M. SPINER
(Moscow)

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A problem of point explosion in an almost ideal gas is considered. A virial power expansion in terms of the parameter $b\,\rho_0$ representing the product of the gas density and the "internal volume" of the molecules, is used as the equation of state. The passage to the ideal gas problem is considered. It is shown that when the motion is adiabatic, then for arbitrarily small values of $b\,\rho_0$, a region of dimension $r \sim r_\Phi \sqrt{b\rho_0}$ exists near the focus of explosion, in which the dimensionless velocity profile differs from the ideal gas profile ($V(\lambda)$ in particular is found to be a nonmonotonous function of the coordinate). Further it is shown that, in contrast to the adiabatic motion, a uniform passage to the case of an ideal gas exists in a heat-conducting gas.

1. When a strong explosion takes place, the character of the motion of the substance depends essentially on its equation of state. In the case of a strong point explosion the motion is found to be self-similar for a wide class of equations of state. Such a motion was studied originally for the case of an ideal gas; subsequently, examples of solutions for the explosion problem were given for certain real, thermodynamically imperfect media [1 – 3]. It should however be noted that the study of explosions in the media differing from the ideal gas did, as a rule, involve empirical equations of state, which only describe the behavior of the medium satisfactorily in a certain, limited interval of densities. Almost every one of these empirical equations was found to be incorrect in the low density region and, in the limit when $\rho \to 0$, it either did not reduce to the equation of state for the ideal gas, or yielded an incorrect first term of the so-called virial expansion for the pressure in powers of density.

As we know from the statistical physics, at low densities the equation of state can be written in the form [4]

$$p = \rho R T [1 + \rho B(T) + \rho^2 C(T) + \ldots] \tag{1.1}$$

where $B(T)$ and $C(T)$ are virial coefficients which can be determined if the molecular interaction potential is known. In the high temperature range, considerably exceeding the critical temperature of the substance (this case will be considered below), the coefficients $B(T)$ and $C(T)$ tend to constant values equal to b and $^5/_8 b^2$ respectively. The quantity $(mb)^{1/3}$ is of the same order as the radius of action of the intermolecular repulsion forces, therefore for gases we have $b\rho \ll 1$. It can easily be verified that in the limit when $\rho \to 0$, the equations of state considered in [2] do not transform to an expression of the form (1.1). Therefore the question of distribution of the hydrodynamic variables in the low density region in a medium perturbed by a strong explosion, remains unsolved.

Below we consider a strong point explosion in an imperfect gas of initial density ρ_0 satisfying the inequality $b\rho_0 \ll 1$. At first we assume that the initial motion of the perturbed medium is adiabatic. It shall be shown that in such a problem a uniform limiting passage to the case of an ideal gas ($b \equiv 0$) does not take place. Actually, at the center of explosion (where, as expected, the equation of state for the medium approximates to the equation of state for the ideal gas most closely) a region exists, for arbitrarily small values of $b\rho_0$, the size of which is $r \sim r_2 \sqrt{b\rho_0}$ (where r_2 is the radius of the shock wave front) and in which the velocity distribution differs from that in the ideal gas.

The change in the velocity profile near the center of explosion in an almost ideal gas is not necessarily physically observable. It should also be kept in mind that in the nonadiabatic case, the equations of motion behave differently at $b\rho_0 \to 0$.

Below we show that when the transfer of heat by thermal conduction is taken into account, then the solution of the problem under consideration reduces smoothly to that for the ideal gas. Moreover it can easily be shown that, when the condition that the motion is adiabatic is replaced by the condition that it is isothermal, $\partial T/\partial r = 0$ and no rearrangement of the velocity profile takes place during the limiting passage from the almost ideal to the ideal gas case. The examples considered below show just how sensitive the self-similar solution of the problem of a strong explosion is to arbitrarily small changes in the equation of state, and the consequent necessity for a careful choice of the thermodynamic model of the medium.

2. Let us consider the adiabatic motion of the gas. The properties mentioned above appear even when the first term of those contained in the brackets in (1.1) is taken into account, and therefore it is sufficient to consider the equation of state in the form

$$p = \rho RT(1 + b\rho) \tag{2.1}$$

The heat capacity of the gas is assumed to be constant. By introduction of the self-similar variables

$$\lambda = r\left(\frac{\rho_0}{Et^2}\right)^{1/5}, \qquad g(\lambda) = \frac{\rho}{\rho_0}$$

$$V(\lambda) = \frac{vt}{r}, \qquad \Pi(\lambda) = \frac{pt^2}{\rho_0 r^2}$$

where E is the energy of explosion, the problem of motion of the medium can be reduced to two algebraic relations (the energy and adiabatic integrals [1]) and one first order ordinary differential equation [1 – 3]. The complete system of equations is

$$\Pi\left[{}^2/_5 - (\gamma + \alpha g)V\right] + {}^1/_2\, gV^2\left({}^2/_5 - V\right)(\gamma - 1 + \alpha g) = 0 \tag{2.2}$$

$$(\lambda/\lambda_2)^5\, \Pi\left({}^2/_5 - V\right) = k(\gamma - 1 + \alpha g)\, g^{\gamma-1} \exp(\alpha g) \tag{2.3}$$

$$\frac{dV}{dg} = \frac{\left({}^2/_5 - V\right)\left\{\left[V(\gamma + \alpha g) - {}^2/_5\right]\left[2 - V(3\gamma - 1 + 3\alpha g)\right] - 3\alpha g V^2\right\}}{g\left\{4\left(V + {}^1/_5\right)\left[V(\gamma + \alpha g) - {}^2/_5\right] + {}^6/_5\left({}^2/_5 - V\right)\right\}} \tag{2.4}$$

$$\alpha = b\rho_0(\gamma - 1), \qquad k = \frac{16\exp(\alpha g_2)}{g_2^{\gamma-1}(\gamma - 1 + \alpha g_2)}$$

where γ is the ratio of specific heats at $\rho = 0$, and the subscript 2 denotes the quantities directly behind the shock wave front. Taking into account the smallness of the parameter α, we can write the boundary conditions at the shock wave in the form

$$g_2 = \frac{\gamma+1}{\gamma-1}\left(1 - \frac{2\alpha}{\gamma-1}\right), \qquad V_2 = \frac{4\,(1+\alpha/(\gamma-1))}{5\,(\gamma+1)}, \qquad \Pi_2 = \frac{2}{5}V_2 \qquad (2.5)$$

The constant λ_2 must be defined from the condition of equality of the total energy of the medium and the energy of explosion E. The ratio of specific heats γ, which in this case must not be confused with the adiabatic exponent, varies within $1 < \gamma \leqslant 5/3$.

Figure 1 depicts the plane of integral curves of (2.4). Only that part of the phase plane of the gV variables is shown which corresponds to the physically realizable states of the medium. Arrows of the trajectories indicate the direction of decreasing λ. It is expedient to eliminate the parameter α from (2.4) by making the substitution $g_* = \alpha g$ (the parameter obviously remains in the integrals (2.1) and (2.2) and in the boundary conditions (2.5)). The singular point A $(0, 2/5)$ is a saddle point with the asymptotes $g_* = 0$ and $V = 2/5$. The singular point B $(0, 2/5\,\gamma^{-1})$ is a node and corresponds to the center of symmetry $\lambda = 0$. The characteristic directions at B are straight lines $g_* = 0$ and $V = 2/5\,\gamma^{-1}(1 - g_*\gamma^{-1})$. The differential equation (2.4) written in the vicinity of the point B differs from the corresponding equation for an ideal gas by the additional term $3V^2 g_*$ appearing in the numerator of the right-hand side. The addition of such a term causes, as we know from [5], a rotation of the whole plane of integral curves near the singular point through a certain angle (in the present case the angle of rotation is arctg γ^{-1}). As a result of this rotation, the dimensionless velocity $V(\lambda)$ approaches its asymptotic value of $2/5\,\gamma^{-1}$ at the center of symmetry from below, i.e. the dimensionless velocity profile ceases to be monotonous (in the case of ideal gas the profile is monotonous and has the minimal value of $2/5\gamma^{-1}$).

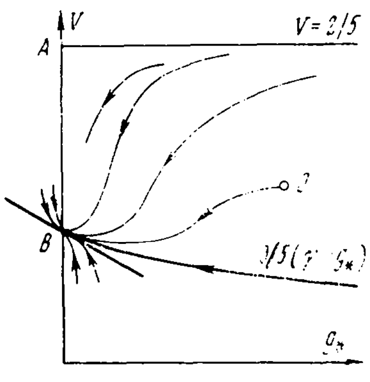

Fig. 1

The transformation from the phase plane $g_* V$ to the physical space is carried out by means of the integrals (2.1) and (2.2). For the velocity $V(\lambda)$ near the center of symmetry we obtain the following asymptotic formula:

$$V(\lambda) \sim 2/5\,\gamma^{-1} - a\lambda^{3(\gamma-1)} \qquad (2.6)$$

where $a > 0$ is a constant (in the case of an ideal gas we have [1] $V(\lambda) \sim 2/5\,\gamma^{-1} + c\,\lambda^{(2\gamma+1)/(\gamma-1)}$, $c > 0$). By taking into account the second term in the expansion (2.6) we can show, that the function $V(\lambda)$ reaches its minimum value at

$$\lambda = \left[\frac{3a}{(2\gamma+1)c}\right]^{\text{[illegible]}}$$

and the value of $V(\lambda)$ at its minimum differs from its limiting value $2/5\,\gamma^{-1}$ by a quantity of the order of $\alpha^{(2\gamma+1)/2(\gamma-1)}$.

When the parameter α defining the nonideal character of the medium tends to zero, the velocity asymptotics near the center do not become the ideal gas asymptotics. However, the interval of values of the variable λ in which the behavior of $V(\lambda)$ is "non-ideal", tends to zero as $\sqrt{\alpha}$ when $\alpha \to 0$. The asymptotic formulas for the dimensionless density and pressure near the center of explosion coincide (unlike those for the velocity) with the corresponding formulas for the ideal gas when $\alpha \to 0$.

3. We consider a strong self-similar explosion in an ideal, heat-conducting gas whose equation of state is (1.1). Let us introduce the self-similar variables

$$\lambda = \frac{r}{r_2}, \qquad V(\lambda) = \frac{\gamma+1}{2D} v(r, t)$$

$$g(\lambda) = \frac{\gamma-1}{\gamma+1} \frac{\rho(r, t)}{\rho_0}, \quad \theta(\lambda) = \frac{R(\gamma+1)^2}{2D^2(\gamma-1)} T(r, t)$$

$$r_2 = \left(\frac{\sigma E t^2}{\rho_0}\right)^{1/5}, \qquad D = \frac{2}{5} \frac{r_2}{t}$$

where σ is a pure number defined from the condition of equality of the energy of explosion and the total energy of the perturbed medium. The requirement that the motion be self-similar imposes restrictions on the dependence of the thermal conductivity on the temperature. By [1] we must set $\varkappa = \varkappa_1 T^{1/6}$, where $\varkappa_1 > 0$ is a constant.

Taking into account the fact that the problem considered here has an energy integral [2], we can use the gasdynamic equations to derive the following system of ordinary differential equations (a prime denoting differentiation with respect to λ)

$$V'g[2V - \lambda(\gamma+1)] + g'\theta(\gamma - 1 + 2\alpha g) + \theta' g(\gamma - 1 + \alpha g) = {}^3/_2 (\gamma+1) Vg$$

$$V'g + g'\left(V - \frac{\gamma+1}{2}\lambda\right) + 2\frac{Vg}{\lambda} = 0 \tag{3.1}$$

$$B\theta^{1/6}\theta' = \frac{12}{\gamma+1}(\gamma - 1 + \alpha g) Vg\theta - 6\left(\lambda - \frac{2V}{\gamma+1}\right)(\theta + V^2) g$$

$$B = \frac{6\varkappa_1 2^{1/6} (\gamma-1)^{13/6} (\gamma+1)^{-4/3}}{(\sigma E)^{1/3} R^{7/6} (0.4\rho_0)^{2/3}}, \qquad \alpha = b\rho_0(\gamma+1)$$

The system (3.1) must be solved under the following boundary conditions:

$$\frac{\gamma+1}{\gamma-1}\left(1 - \frac{2}{\gamma+1} V_2\right) g_2 = 1, \qquad \left(1 - \frac{2}{\gamma+1} V_2\right) = \frac{\gamma-1}{\gamma+1}\theta_2\left(1 + \frac{\alpha}{\gamma-1} g_2\right)$$

$$B\theta_2^{1/6}\theta_2' = \frac{12}{\gamma+1}(\gamma + \alpha g_2 - 1) V_2 g_2 \theta_2 - 6\left(1 - \frac{2}{\gamma+1} V_2\right)(\theta_2 + V_2^2) g_2$$

and the symmetry condition $V(0) = 0$.

According to the results of Sect. 2, the motion of gas near the center of symmetry is of particular interest, i.e. in the region in which the adiabatic motion gives rise to an additional term in the asymptotic formula, namely the term preventing a smooth passage to the ideal gas formulas. Using (3.1) to compute the asymptotics of the gasdynamic variables as $\lambda \to 0$, we obtain the following formulas for the dimensionless velocity, density and temperature:

$$V = \frac{(\gamma+1)^3(\gamma - 1 + \alpha g_0) g_0}{5B\theta_0^{1/6}(\gamma - 1 + 2\alpha g_0)} \lambda^3$$

$$g = g_0\left[1 + \frac{3(\gamma - 1 + \alpha g_0)}{B\theta_0^{1/6}(\gamma - 1 + 2\alpha g_0)}\lambda^2\right] \tag{3.2}$$

$$\theta = \theta_0\left[1 - \frac{3g_0}{B\theta_0^{1/6}}\lambda^2\right]$$

Here g_0 and θ_0 denote the dimensionless density and temperature at the center of symmetry (when $\gamma = 1.4$ and $B = 1.48$, we have, according to [2], $g_0 = 0\ 052$ and $\theta_0 = 2.04$).

Formulas (3.2) show that when the parameter α tends to zero, the limiting expressions for an almost ideal gas pass smoothly to the corresponding expressions for the ideal gas. Thus, by taking the heat conduction into account we can obtain not only a finite value for the temperature at the center of explosion, but also a correct limiting passage from the imperfect to the ideal gas case. Corrections to the solution of the problem of explosion in an ideal thermally conducting gas caused by the nonideal character of the medium can be easily obtained from the system (3.1).

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CHARACTERISTICS OF TRAJECTORIES OF A FIELD GENERATED BY RANDOMLY DISTRIBUTED SOURCES

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A. A. ALEKSEEV and B. M. STRUNIN
(Moscow)

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We investigate the statistical characteristics of trajectories of a (scalar, vector, tensor) random field σ generated by independently distributed sources. Formulas obtained for the trajectory of a component σ of such a field along an arbitrary straight line x_0, define the mean values of the following characteristics (see Fig. 1): $l^+ = l^+(\tau)$ and $l^- = l^-(\tau)$ are the distances between two neighboring upcrossings and downcrossings of the level τ; $\xi = \xi(\tau)$ is the duration of an upwards excursion across τ, while $\lambda^+ = \lambda^+(\varkappa)$ and $\lambda^- = \lambda^-(\varkappa)$ are distances separating the consecutive points on the trajectory possessing the same first order derivative with respect to $\varkappa$, with positive and negative curvature, respectively. The formulas differ from those given by the general mathematical theory of trajectories of stationary random processes; they can be applied in practice to obtain the characteristics of traject-

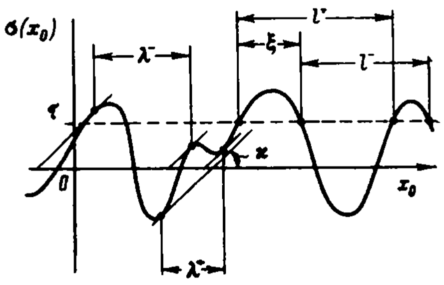

Fig. 1